Review of the Icelandic Wage Price Index

Report to the Wage Statistics Committee
Government of Iceland

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Introduction

Statistics Iceland compiles the Icelandic Wage Price Index (IWPI) from a survey of employer payrolls. The item whose price movements the IWPI summarizes is the employee-hour worked, and the price concept for that item is monetary compensation per hour worked.[2] According to employer associations and unions, the IWPI is rising faster than specified in Icelandic labor agreements, in a labor force that is around 80 percent covered by such employer-union agreements.[3] This divergence has led employer and labor organizations to question Statistics Iceland's narrow specification, "matched models" methodology for the IWPI, and propose that some form of averaging of wage levels be applied before calculating wage change, following the practice of other Nordic countries.[4]

This consultancy has been tasked to

- Review and evaluate the statistical methods used by Statistics Iceland in the IWPI;
- Assess and evaluate potential errors and biases in the IWPI in the context of its intended use;
- Review the structure of the IWPI in the combined context of data, methods, and potential biases; and
- Recommend improvements.

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[1] An earlier version of this report was presented to the Wage Statistics Committee on November 1, 2018. This version corrects typographical errors, clarifies language, and adds explanatory material based on the discussion in that meeting, but changes no conclusions or recommendations.
[2] The IWPI item price thus excludes non-monetary forms of compensation such as employers' social contributions in behalf of employees (for example, contributions to retirement benefits). The labor cost index or price index for total compensation of employees includes nonmonetary forms of compensation—employer purchases on behalf of employees in lieu of monetary compensation—as well as monetary compensation in summarizing the price changes of labor services. As well as the IWPI, Statistics Iceland has published an index on total wages based on the labor cost index, which is part of the Eurostat Labor Cost Index system.
[3] Table 3 in Joint Committee on Payroll Information and Economic Assumptions for Collective Agreements (2016).
[4] SA Confederation of Icelandic Enterprise (2018).

Summary of Findings

This report supports the fundamental methodology for the IWPI documented in Statistics Iceland (2018b) as an index of the prices of narrowly-specified labor services but suggests that there could be a labor quality effect inherent in the IWPI's otherwise commendably granular employee-hour-worked index item specification. If quantitatively significant and plausible, this should be excluded from the IWPI's estimate of wage change. For example, when calculating the wage relative of the employee record, the wage in the numerator refers to an employee with one month more experience, in total and with the current employer, than the wage of the employee in the denominator, notwithstanding their reference to the same individual. To examine this, it suggests a quality adjustment method using hedonic wage equations to apply to the IWPI's employee record (payroll) wage source data before indexes are calculated at the first level of aggregation, the industry-by-occupation "cell." The proposed hedonic equation sets hourly wage as a function of three key time-incrementing labor quality variables available on Statistics Iceland's ISWEL[5] database—educational attainment, total experience (proxied by age), and experience with current employer—as well as other, non-time-incrementing employee characteristics.

To examine the remedy proposed by the IWPI's critics, the report also considers an alternative methodology for measuring wage change used by other Nordic statistical agencies[6] as well as other countries collecting wage data from payroll sources,[7] which is to calculate ratios between periods of average wages of some form rather than the IWPI's Törnqvist "matched models" (actually, "matched employees") index at its "cell" level of aggregation. The type of average chosen for this calculation can help to shed light on what a ratio of average wages methodology

[5] Icelandic Survey on Wages, Earnings and Labour Costs.

[6] See, e.g., Statistics Denmark (2018).

[7] The Employment Cost Index, a quarterly labor compensation series produced by the Bureau of Labor Statistics (BLS), has a subtly different item definition from the IWPI—the job rather than the employee. The item specification for the job is defined by a detailed occupational classification and a "job level" within that classification. The job "level" is determined as an item specification when establishments are initiated into the survey sample, very much like the approach BLS uses for its goods and services price indexes—the CPI and PPI. The "job leveling" process requires significant field staff training and "calibration meetings" to minimize as much as possible variation in the "level" that would be assigned to the same job by different field staff. A sample of employee records is then taken for each job specification and hourly wage and nonwage compensation data are averaged within that detailed cell. *By implication, the U.S. Employment Cost Index eliminates compensation changes due to change of job level from its estimate of labor cost change.* See U. S. Bureau of Labor Statistics (2013, 2017). The BLS also produces a monthly payroll-based hourly earnings series from the Current Employment Statistics (CES) survey, which is based on an unemployment insurance sample frame. The CES collects from each sample establishment only total payroll, employee count, and hours paid. Because it does not collect employee record data, it has none of ISWEL's employee characteristics information. Within each industry by occupation cell, BLS calculates percentage changes in average earnings from a sample matched by employer between successive months. See U. S. Bureau of Labor Statistics (2011), and p. 6 of that source for description of matched sample estimates. *It should be noted that, although the BLS average earnings series is employed by many users of statistics as a high frequency, monthly indicator of wage change, BLS considers the quarterly Employment Cost Index to be its definitive price measure for labor services due to its narrower and more homogeneous "job" (occupation and level) specification for the labor services priced.*

would do differently from the current IWPI methodology. The Törnqvist formula Statistics Iceland uses for month to month change in wages can be written as a ratio of share weighted geometric means of employee wages within each cell. Thus, choosing a ratio of those specific share weighted geometric averages is identical with the IWPI if there are no hires or separations between the months compared, so that all wage records are matched between the two periods. The ratio of weighted geometric averages approach therefore differs from the IWPI only by the appearance of the wages of hires and the disappearance of the wages of separations in the numerator geometric average. [8]

It is possible to factor the ratio of weighted geometric averages into the IWPI's Törnqvist wage index for employees neither hired nor separated in the current month, and a "change of assortment" factor depending, in part, on the ratio of the average wages of hires in the current month, if any, with the average wages in the previous month of employees separating in the current month, if any. When separating employees are more experienced (in total) and thus earn higher wages than hires, the latter ratio, when present, might be seen as a very rough indicator of the effect of the slope of the pay scale with respect to experience, the latter being a key time incrementing labor quality variable. In other circumstances, however, ratio of averages does not reflect the ongoing labor quality effect of experience. When the items in a cell cannot be fully matched between periods, ratio of averages indexes are at least as affected by quality heterogeneity in the index cell, and as much in need of quality adjustment to the prices before the averages are calculated, as indexes of items matched on some, but not all price determining characteristics. When a match on all characteristics *is* possible, matched models is clearly superior. This is a key reason why international recommendations favor quality adjusted index numbers over unit values.

Economists would classify the hires and separations indexing problem as analogous to the new and disappearing goods problem in goods and services price indexes. This is a difficult analytical and empirical issue to solve, and it is not clear that the assortment adjustment factor of the ratio of geometric averages cell index conforms to their "reservation price" concept for handling the change in assortment problem.

Use of arithmetic average wage (unit value) cell indexes is not recommended for the IWPI, as the employee record collections in the IWPI cells are unlikely to be sufficiently homogeneous to defend using this formula.

[8] It is worth noting that additional employee record information on the wages of hires does affect the "matched models" IWPI, but with a four-month lag, as Statistics Iceland's practice is to introduce new employee records into the index after the first three months of wage reports. Separations are dropped from the index after three months of dormancy. Hires can enter the "matched models" IWPI no sooner than when there are two months of data.

Methodology Review and Assessment

Data sources

From Statistics Iceland (2018b), the IWPI database is fed from a payroll survey called the ISWEL that is broadly similar to other Nordic wage data sources. It is designed to provide payroll information on the population of employers having at least 10 employees, in economic activities covering about 74 percent of Iceland's total national employment in 2017 of 197,600. Employers are sampled from these strata, and the entire payroll of each sampled employer is captured monthly. The survey generates about 70,000 employee records per month or 100,000 employee instances per year, respectively, about 35 and 50 percent of the 2017 labor force. The ISWEL complies with European Economic Area guidelines on the concepts underlying the items collected.

- Assessment: The ISWEL's coverage appears to be a robust foundation for wage measurement.

The variables in the record layout of the ISWEL comprise the entries in the table below, which indicates variables having relevance to various aspects of compensation measurement with color shading.

- The L and V variables identified by the survey are highlighted with green and measure wage compensation, whose hourly rate is tracked in the IWPI.
- The K type variables identified by the survey are highlighted with yellow and measure non-wage compensation, they would be included in the more comprehensive total compensation or labor cost index.
- *Those S variables highlighted in orange are measures or proxies for labor quality.* The F, S, and T variables with no color highlight are controls that would be used to distinguish labor quality from labor price change.
- The F and S variables highlighted in light gray can be used to construct indicators of an employee record experiencing a job change, because the employee changed firms, changed occupations, or both; along with the un-highlighted control variables, the job change indicator would be used to distinguish labor quality from labor price change.

- Assessment: The ISWEL's record layout includes information allowing quality adjustment of employee wages should there be a shift in the employee specification; the survey data content appears to be a good foundation for wage measurement.

Aggregation and basic calculation architecture

From Statistics Iceland (2018b), the IWPI aggregates employee record wage information at the first stage by calculating Törnqvist month to month indexes for industry by occupation "cells." Appendix A provides a description of the Törnqvist index number and summary of its

	Record Layout of the ISWEL Survey: Identifiers and Labor Quality Indicators[9]		
No.	Item designation	Label	Notes
1	Company ID No.	F	Between-firm change of job
2	Municipality	F	
3	Economic activity (5 digit code: NACE + 1)	F	
4	Employee ID No.	S	Principal record ID
5	Month and year of birth (Age = Current date – Month and year of birth)	S	Labor quality – Total experience
6	Sex	S	
7	Union	S	
8	Pension fund	S	
9	Education code (1 digit code: ISCED)	S	Labor quality – Education
10	Occupation code (5 digit: ISCO88 + 1)	S	Between-occupation change of job
11	Length of service (Date of employment) = Current date – Date of employment)	S	Labor quality – Experience with firm
12	Proportion to full-time employment	S	
13	Pay period	S	
14	Contractual working hours	S	
15	Annual leave entitlement percentage	S	
16	Annual leave arrangement	S	
17	Agreement	S	
18	Wage group	S	
19	Wage level	S	
20	Basic wages and salaries	L	
21	Normal hours	V	
22	Additional payments	L	
23	Expenses payments	L	
24	Bonus payments	L	
25	Piecework payments and output work	L	
26	Shift premium	L	
27	Hours with shift premium	V	
28	Overtime pay	L	
29	Overtime hours	V	
30	Sickness pay	L	
31	Sickness hours	V	
32	Lump sums and special payments	L	
33	Committee or management payments	L	
34	Payments for transport	L	
35	Fringe benefits	L	
36	Other payments	L	
37	Remuneration paid for leave	L	
38	Pension fund contribution	K	
39	Social security tax	K	
40	Sickness fund payment	K	
41	Vacation (union) housing fund fee	K	
42	Science fund / continued education	K	
43	Other labour costs	K	
44	First day of payment period	T	
45	Final day of payment period	T	
	F – Data on enterprises S – Information on employees [of which, labor quality] L – Wage items V – Details on working hours K – Labour cost T – Dating of pay record – Change of firm or change of occupation status (change of job)		

[9] See Statistics Iceland (2018b), Appendix 5.

properties. The cell indexes are then chained into a set of cell monthly time series.[10] The cell monthly time series are then aggregated to the industry, occupation, and all items levels using the Laspeyres formula, whose annual weights are updated each year and is annually chained.

- Assessment: The aggregation and basic calculation architecture of the IWPI is sound.
  - The Törnqvist cell level aggregation formula is known to be superlative—exact for an underlying labor cost function aggregator that can be expected to be very accurate in accommodating substitution between employees by employers as relative wages change.
  - The Laspeyres aggregation formula from cells to industries, cells to occupations, and cells to all items is additive and simplifies interpretation of the different published presentations of the index by these breakdowns.

Quality change methodology

Statistics Iceland follows standard conventions in linking out non-comparable substitutions triggered by changes in infrequently incrementing employee characteristics (items 1-3, 6-8, and 12-17 in Table 1).

- Assessment: Statistics Iceland's standard approach is sound for episodic changes detected in infrequently incrementing characteristics.

There are three labor quality correlates on the employee records in Statistics Iceland data sources that display more frequent time-incrementing behavior:

– Total experience, for which SI data sources contain Age (current month less Date of birth, item 5 in Table 1) as a useful proxy.[11]
– Experience with employer, which is the difference between the end-date of the current index month and the date of employment with the current employer (the latter labelled as Length of service, item 11, in Table 1).
– Education (item 9 in Table 1), which is coded in Statistics Iceland sources by ISCED[12] level of attainment.

[10] Statistics Iceland (2018b) reports on an internal examination of possible chain drift at the cell level of the above described methodology and concludes that it is negligible.
[11] Age is a proxy for Total experience because it includes years prior to entering into the current employment arrangement. In Iceland, the initial age of employment is thought to be around 14, so one approach to tailoring Age more closely to Total experience is to subtract 14 from it. However, there presumptively is some variation in age of entry into the labor force, so that adjustment of the employee record does not completely deal with this shortcoming of Age for the purpose here. Nevertheless, this report deems the lack of employee record information on date of entry into the labor force a minor problem and considers Age a serviceable proxy for Total experience.
[12] International Standard Classification of Educational Attainment, see UNESCO (2012).

Precedent for using Age and Education as labor quality indicators goes at least as far back as Victor Fuchs (1964). Education increments periodically in early career, while Total experience and Experience with employer increment every month over the entire career. Movements in monetary compensation *over time* that are related to any of these—at either the individual or cell (industry by occupation) level—are labor volume movements. As they are not price changes, their effect should be removed from the monthly cell relative via quality adjustment of the individual employee monthly wage relatives. Statistics Iceland's current quality adjustment protocol either deals with these time incrementing quality changes partially for education [13] or does not address them at all for the experience variables.

- Assessment: Statistics Iceland's current quality adjustment methodology does not account for change in the regularly incrementing labor quality characteristics on the ISWEL employee record; this could result in persistent upward bias in the IWPI's estimates of price change. [14]

To quality correct for education and experience, Statistics Iceland could check the explanatory power of the three quality indicators for wage by estimating a cross-sectional wage equation on each month of source data, as a function of these as well as additional controls that are measured in the survey and available in the record layout for each employee.[15] Statistics Iceland already has a running start on this based on its Gender Pay Gap Analysis, which showed that within each year of ISWEL data, Education, Age, and Experience with employer had positive and statistically significant coefficients.[16] The parameter estimates of this model would then be used to quality adjust the IWPI for each employee's accumulation of education and experience within the current architecture of the index, which uses the Törnqvist index formula to aggregate month to month wage changes from employee records within industry × occupation *cells*, which are subsequently aggregated using a Laspeyres formula to higher level categories. Appendix B describes in general how this methodology would work.

Is "relative of average wages" a simple solution to the labor quality change issue?

Appendix C shows that if there are no hires or separations in a comparison of two months of ISWEL payroll data within a given industry × occupation cell, and if the averages being compared use a specific weighted geometric average, then the narrow specification Törnqvist cell index of the IWPI and the ratio of cell averages approach are algebraically identical and will

[13] While Education is a time-incrementing variable, at least in early career, Statistics Iceland partly controls for this implicitly because granting of a degree or other educational certification tends to be close in time to change of occupation, which is a specification change and thus linked out of the estimate of price change.

[14] Statistics Iceland's preliminary exploration of quality adjusting the IWPI's Törnqvist cell relatives for incrementing of experience and education at the suggestion of the consultant concluded that its monthly impact is quantitatively small. This should be followed through, however. Monthly effects will indeed be quite small but can compound to non-negligible adjustments over longer periods.

[15] Economists call these "hedonic equations," a term attributable to Court (1939), the first known cross sectional regression study of this type.

[16] Statistics Iceland (2018a).

yield identical price change results. So the relative of averages approach only makes a difference when there are hires and separations, in which case the relative of averages becomes the product of the IWPI Törnqvist index of the wages of employees present in both the current and previous month (a "matched models" index) with an adjustment factor that reflects the change in the assortment of employees within the cell arising from hires and separations.

Hires and separations in the labor cost measurement context are formally analogous to new and disappearing goods in the commodity price measurement context. The long accepted conceptual approach to this problem is to set the unobserved previous period price of the new item, or the unobserved current period price of the disappearing item, at their respective reservation prices. The reservation price is the price at which the quantity of hours of labor services demanded by the purchasing employer is zero.[17] Though analytically well established, the reservation price approach has been difficult to implement in practice.

Another approach to this problem, well established in price survey sample design theory and practice, is to stratify the sample of employees into *homogeneous* elementary classes. This is the intent of the stratification of the IWPI sample into industry by occupation *cells*. If the class is homogeneous, then a ratio of average cell prices—a ratio of cell unit values—can be justified.[18] If the ISWEL data in the IWPI cell are not homogeneous, unit values are not recommended, because comparison of cell averages will not be comparing the prices of similar items between the current and previous periods, and thus will not produce good measures of price change.

The most widely used practice in official index numbers, including the IWPI, is that the unobserved previous period price of a new item (hires) is that item's current period price divided by the price index relative of the cell where it resides, where the relative is based on items in the cell having both current and previous period prices; that is, is a "matched models" relative. Similarly, the unobserved current period price of a disappearing item is its previous period price times the same "matched models" cell relative.

A synthesis of the three approaches to change in assortment is to use a ratio of averages of *quality adjusted* prices within each industry $\times$ occupation cell. The argument for this is that item by item quality adjustment converts the dissimilar items in the assortment available in any given period into similar items, justifying the use of a ratio of averages to handle assortment

[17] Fisher and Shell (1972).

[18] See International Labour Office, International Monetary Fund, Organization for Economic Cooperation and Development, Statistical Office of the European Communities (Eurostat), United Nations, and World Bank (2004), paragraph 7.142, p. 127 and Appendix 8.1, paragraph 15, p.149. Although this citation is from the *Consumer Price Index Manual*, it applies with equal force in all index number contexts. Footnote 1, p. xiv of the *Producer Price Index Manual* states "Note that the Manual does not endorse taking unit values over heterogeneous items at this first stage of aggregation; it endorses only taking unit values over identical items in each period." See also paragraph 1.154, p. 29 of the same source at International Labour Office, International Monetary Fund, Organization for Economic Cooperation and Development, Statistical Office of the European Communities (ENurostat), United Nations Economic Commission for Europe, and World Bank (2004).

variations from period to period. This report proposes a quality adjustment methodology that is compatible with the current Törnqvist aggregation framework at the IWPI cell level. Were it applied in the context of a ratio of (weighted geometric) average prices methodology, it could be used in combination with averaging to accommodate changes in the assortment of employees from month to month in employers' workforces. Having taken note of this, however, it seems that after successful quality adjustment the quantitative effect of this incorporation of the previous month wages of separations and current month wages of hires into the monthly quality adjusted price relative would be reduced if not eliminated, and that using a quality adjusted IWPI "matched models" index would be sufficiently accurate.

Recommendation

Statistics Iceland should continue with its current survey and index calculation architecture but explore quality adjusting IWPI employee wage records for experience and education using the hedonic method.

Concluding Remarks

The objective of price indexes is to determine the contribution of change in prices to the change in a value aggregate, with the residual thus attributable to volume (quantity and quality) change. Any error in the price index induces an offsetting error in the corresponding volume index. Notwithstanding the need for accuracy in the direct uses of the wage or labor cost index, the knock-on error in labor volume has consequential downstream effects on labor and multifactor productivity measures. Accurate price measurement requires, at the end of the day, that the prices of the same items be compared. This requirement underlies the international recommendation that a unit value (average price) comparison of the prices of a collection of items only be used when the items in the collection are homogeneous; that is, there should be no un-accounted-for price-determining characteristic whose variation across the collection affects the productivity or utility of use the items in the collection. This criterion is the motivation behind the detailed stratifications of data on which price indexes are based and is the reason international standards for price indexes uniformly prefer index number rather than unit value comparisons of price change for collections of items, unless the homogeneity of the collection is assured. In the interest of adhering to this recommendation, the IWPI methodology stratifies down to the employee record, which holds all employee characteristics that tend to change infrequently fixed, using link out-link in methods to control for quality change when these characteristics do change.

In some cases, meeting this homogeneity, "like-with-like" criterion for items or collections compared can become impossible to achieve by stratification alone. This has led analysts to seek "continuous stratification," "quality adjustment" methods to deal with these cases, and the prevalent methodological approach is the so-called hedonic method of Court (1939). The hedonic method requires collection of data on price-determining characteristics, along with the prices themselves, that are used to predict the contribution to the proportional difference between the prices of two items of the difference in their price-determining characteristics, and thus their difference in quality.

In the IWPI, there are employee characteristics—experience in particular—whose time-incrementing behavior could produce this kind of inhomogeneity of the employee records within its cell strata, because it is not possible to directly compare the wage of an employee in the present period whose experience is the same as the experience the same employee had in the previous period—experience will always be incremented by 1. As a result, the IWPI will not only reflect change in the levels of company wage scales, which are labor price changes, but also quality creep along those wage scales, which are labor volume changes. As described in this report, the importance of this issue can be explored by estimating hedonic equations for each cell in each month and quality adjusting month to month employee wage relatives for time-incrementing variables such as experience before calculating the Törnqvist month to month cell indexes.[19] Although Statistics Iceland believes that its initial exploratory data analysis suggests the impact of the experience effect is limited, follow though is warranted, as it is a concern with using the employee record as the item in a wage or labor cost index. A significant effect suggests implementing a robustly estimated hedonic quality adjustment to the IWPI in the short run and, possibly, adding promotion and seniority increase items on the ISWEL employee record in the longer run.

In the consultant's view, averaging the wages in IWPI cells prior to forming relatives for higher level aggregation is of academic interest provided the averages are weighted geometric averages compatible with the IWPI's Törnqvist month to month cell index, and that the prices in the averages are quality adjusted. However, the conceptual underpinnings of the measure of the assortment effects of hires and separations that this would introduce into the IWPI are not well understood, though successful quality adjustment can be seen as a way of eliminating inhomogeneities in the IWPI cell, which would support the ratio of quality adjusted geometric weighted averages from another angle. However, the additional effect of this is mostly in the variance of the index rather than in the expected level and change of the index, and to my knowledge, the exact form of the assortment adjustment implied by the Translog functional form underlying the Törnqvist index has not yet appeared in published research on the new and disappearing items problem. It is therefore not yet ready for index production. Arithmetic average wage (unit value) cell indexes are not recommended.

[19] It is worth emphasizing that there is precedent for implementing the hedonic method for a closely analogous problem in a key official price series: correcting for aging bias in the US CPI rent index. Aging bias in the CPI rent index is essentially the same problem as controlling for the experience effect in the IWPI. The rent component of the US CPI has substantial weight; thus the Bureau of Labor Statistics' hedonic aging bias adjustment influences a nontrivial part of that headline series. See Campbell (2006) and Randolph (1988).

Appendix A. The IWPI Törnqvist Elementary Aggregator

To describe the properties of the Törnqvist index used at the cell level of the IWPI, we begin with a microeconomic model. Define the employee record by the following subscripts:

Industrial activity	- $i = 1, \cdots, n_Y$
Occupation	- $j = 1, \cdots, n_O$
Employee ID	- $k = 1, \cdots, n_E$

Employers represented in IWPI industry × occupation cell ij in month t are treated as an aggregate employer minimizing the cost of producing an $m \times 1$ output vector y_{ij}^t with respect to hours worked, where each employee k works hours h_{ijk}^t and earns wage w_{ijk}^t per hour.

Setting up a vector format to keep notation uncluttered, the cell output vector is

$$y_{ij}^t = \left[y_{ij1}^t, \cdots, y_{ijm}^t\right]',$$

the cell wage vector is

$$w_{ij}^t = \left[w_{ij1}^t, \cdots, w_{ijn_E}^t\right]',$$

and the cell hours vector is

$$h_{ij}^t = \left[h_{ij1}^t, \cdots, h_{ijn_E}^t\right]'.$$

Since all of the following refers to a given IWPI cell ij, we suppress the ij subscript—leaving it understood—to further simplify notation. The cell labor cost function is

$$C^t(y^t, w^t) = min_{h^t}\{w^{t\prime}h^t \colon (y^t, w^t) \text{ is feasible}\}.$$

If $C^t(y^t, w^t)$ has the translog functional form

$$\begin{aligned} lnC^t(y^t, w^t) = {} & \alpha_0^t + \alpha_1^{t\prime} lny^t + \alpha_w^{t\prime} lnw \\ & + \tfrac{1}{2} lny' \Lambda_{yy} lny + \tfrac{1}{2} lnw' \Lambda_{ww} lnw + lny' \Lambda_{yw} lnw \end{aligned}$$

where Λ_{yw}, Λ_{yy}, and Λ_{ww} do not vary over time and Λ_{yy}, and Λ_{ww} are symmetric matrices, then results from Diewert (1976) and Caves, Christensen, and Diewert (1982) can be employed to show that the exact index number for wage change in cell ij is the Törnqvist formula

$$\left[\frac{C^s(y^s,w^t)}{C^s(y^s,w^s)}\frac{C^t(y^t,w^t)}{C^t(y^t,w^s)}\right]^{\frac{1}{2}} = T^{st} = e^{lnT^{st}} = \prod_k \left(\frac{w_k^t}{w_k^s}\right)^{\frac{1}{2}(\omega_k^s+\omega_k^t)}$$

where

$$\omega_k^t = \frac{w_k^t \odot h_k^t}{w^{t\prime} h^t}$$

is the share of employee k in the wage bill $w^{t\prime}h^t$ of cell ij, with

$$w_k^t \odot h_k^t = \left[w_1^t h_1^t, \cdots, w_{n_E}^t h_{n_E}^t\right]'.$$

In summary, the Törnqvist index used at the IWPI cell level is *superlative*—it is exact for a *flexible* functional form—the Translog—that can approximate any twice differentiable cost function to the second order. As such, when the price and quantity vectors in the comparison of prices between two periods are close to one another, as when the formula is used for the month to month links of a chain index, it is very accurate in reflecting enterprise substitution behavior between inputs as relative prices change.

Appendix B. Hedonic Quality Adjustment

To develop notation for the quality adjusted wage index, define the employee record by the following subscripts:

Industrial activity	- $i = 1, \cdots, n_Y$
Occupation	- $j = 1, \cdots, n_O$
Employee ID	- $k = 1, \cdots, n_E$
Employee quality attributes	- $l = 1, \cdots, n_A$
Employee demographic attributes	- $m = 1, \cdots, n_D$

The ISWEL employee record will contain the following employee quality attributes, indexed by l:

Age ($l = 1$)
Length of service[20] ($l = 2$)
Education (ISCED level completed) ($l = 3$).

Thus $n_A = 3$ is the dimension of the employee quality attributes vector with $a = (a_1, a_2, a_3)$. Employee sociodemographic attributes, of which there are n_D, are indexed by m, and are used as additional control variables in the wage hedonic to improve the estimates of the effects on hourly wage of the employee quality attributes.

Under the hedonic technique, for every position k in the ij^{th} cell, wage in month t is determined by the same hedonic equation

$$w_{ijk}^t = H_{ij}^t\left(a_{ijk}^t; d_{ijk}^t\right).$$

To unclutter the notation, we will suppress the ij subscript, leaving it understood that the following applies to a given cell ij, and write

$$w_k^t = H^t(a_k^t; d_k^t)$$

providing the market equilibrium price in month t of the labor services provided by each employee k with characteristics $(a_k^t; d_k^t)$. Typically, as in Statistics Iceland (2018a), this equation has a semi-log of order 2 functional form

$$lnw_k^t = lnH^t(a_k^t; d_k^t) = \gamma_0^t + \gamma_d^t{}' d_k^t + \ d_k^t{}' \Gamma_{dd} d_k^t + \gamma_d^t{}' a_k^t + \ a_k^t{}' \Gamma_{aa} a_k^t + d_k^t{}' \Gamma_{da} a_k^t$$

where a_k^t is the set of time incrementing determinants of employee k's wage w_k^t, d_k^t is a set of non-incrementing (mostly demographic) determinants of wage, and Γ_{dd} and Γ_{aa} are symmetric

[20] Reference date of the month – Date of employment (with current employer, from ISWEL).

matrices. Typically, models like this are estimated with a number of zero restrictions on the second order parameters to avoid parameter saturation, particularly within the interaction coefficients matrix Γ_{da} and among the off-diagonal interaction elements of Γ_{aa} and Γ_{dd}. For simplicity, we will not impose such restrictions here, which will be specific to the application of the model.

The quality adjusted log change in employee k's wages, which excludes the effect of wage determining characteristics that increment through time, is then

$$lnw_k^{*t} - lnw_k^{*s} = (lnw_k^t - lnw_k^s) - [(\gamma_d^t + \ d_k^s{}'\Gamma_{da})'(a_k^t - a_k^s) + (a_k^t{}'\Gamma_{aa}a_k^t - a_k^s{}'\Gamma_{aa}a_k^s)].$$

In any given comparison of month t with month s the term in the square brackets will be positive for time incrementing employee characteristics associated with higher wages and thus higher quality.[21] By implication, for each employee log wage relative the log quality adjustment will be negative and the associated quality adjustment factor to the Törnqvist cell index will be less than one. Thus the quality adjusted index will show lower log price change than the unadjusted index.

The quality adjusted Törnqvist index for cell ij is then

$$T^{st} = e^{lnT^{st}} = \prod_k \left(\frac{w_k^{*t}}{w_k^{*s}}\right)^{\frac{1}{2}(\omega_k^s + \omega_k^t)}.$$

Fixler and Zieschang (1992) derive a quality adjusted Törnqvist formula using a more general microeconomic framework than implied by the "repackaging" approach to quality described here, but this approach is in the mainstream of the hedonic literature, is relatively simpler to describe and implement, and still can fully exploit the information in Statistics Iceland's ISWEL data source.

Caveats with the hedonic method

The accuracy of hedonic quality adjustment depends on the robustness of the coefficient estimates on the time incrementing wage determinants: education, age, and experience with current employer. Although the initial recommendation of this report is to estimate the wage equations independently for each month in each industry × occupation cell of the IWPI, some

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[21] The estimates of Γ_{aa} in Statistics Iceland (2018a) imply that in any given month t, the second order part of the quality adjustment is $a_k^t{}'\Gamma_{aa}a_k^t < 0$, and if a_{ijk}^t is incrementing through time so that $a_k^{t+1} > a_k^t$, then $(a_k^t{}'\Gamma_{aa}a_k^t - a_k^s{}'\Gamma_{aa}a_k^s) < 0$. Although $(a_k^t{}'\Gamma_{aa}a_k^t - a_k^s{}'\Gamma_{aa}a_k^s)$ will become numerically larger for more experienced employees, in ISWEL data, this blunts but does not overwhelm the effect of the positive linear component of the quality adjustment term $(\gamma_d^t + \ d_k^s{}'\Gamma_{da})'(a_k^t - a_k^s)$. Thus, experience increases employee wage—more for inexperienced than experienced employees—and the adjustment for month to month change in experience partly offsets the relative increase in employee record wages from month to month.

cells may be inherently small, limiting precision of the estimates. If the hedonic method is implemented, some experimentation with and adjustment of model specification will be necessary both initially and on an ongoing basis. Some cells may have to be combined with others, or the evolution of monthly hedonic coefficients smoothed, with the prior that pay scales will not radically change slope or curvature from month to month. As with the coefficient estimates of any econometric model, there is a risk of omitted variables bias.

A robustness check on the hedonic model-based education- and experience-adjusted IWPI

It may be possible to construct a series to provide a robustness check on the education and experience adjusted IWPI using historical information. Doing this requires, within each IWPI cell each month, grouping employee records into clusters defined by education and experience and calculating the monthly relatives of geometric average wages for clusters having the same education and experience. The cluster relatives would then be weighted together using the Törnqvist formula to obtain the cell relative. These clusters are similar to the "job" items of the BLS Employment Cost Index, where in the IWPI education and experience would play the role of "level" within occupation.[22] The possibility of doing this check depends on whether, in each IWPI cell, there are enough nonempty clusters containing adequate numbers of employee records, where the nonempty clusters (rather than the employees) can be matched between adjacent months.

The average wages for clusters in this index estimator are not as tightly defined as the employee record wage with adjustment for education and experience. Indeed, by definition, cluster pairs between adjacent months representing the same experience level will have no employee wage record information in common at all because of the monthly incrementing of experience in individual employee records. Thus, the clusters will tend to be heterogeneous in employee characteristics other than education and experience that occasionally but do not regularly change as months pass and/or employees are hired and separated, and the ratio of (geometric) averages cluster index will contain some undesired composition effects. That said, significant divergence between the education and experience adjusted IWPI and the cluster-based index over historical periods of 1-3 years, taking account of the measurement issues in the cluster-based index, would suggest rechecking the specification of the hedonic equation in the education- and experience-adjusted IWPI. [23]

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[22] See U. S. Bureau of Labor Statistics (2017), pp. 9-13. As annotated earlier, the BLS "job" index item is defined by a detailed occupation code and a "level" within that occupation code. Wages (as well as nonwage compensation) are averaged to determine the item price. Job "leveling" is described in more detail in U. S. Bureau of Labor Statistics (2013).

[23] In offering this clustering method for checking the robustness of the hedonic quality adjusted IWPI, this report supposes that average non-education and non-experience characteristics, while fluctuating over the short to medium term, are not trending within education and experience clusters over 12-36 month windows, which could be confirmed from the data.

Appendix C. Comparing the IWPI "Matched Models" Cell Index with a Ratio of Average Wages Index at the Cell Level and Dealing with Changes in Assortment

This appendix considers the form of a wage index constructed from averages of employee wage records at the IWPI industry × occupation cell ij. To make the average wage approach compatible with the Statistics Iceland IWPI aggregator formula, we consider weighted geometric average prices where the employee weights are determined using each employee's time averaged share of the total earnings of the cell

$$\bar{\omega}_{ijk}^{st} = \tfrac{1}{2}\left(\omega_{ijk}^{s} + \omega_{ijk}^{t}\right).$$

with

$$\omega_{ijk}^{s} = \frac{w_{ijk}^{s} h_{ijk}^{s}}{\sum_{k \in cell_{ij}^{s}} w_{ijk}^{s} h_{ijk}^{s}}.$$

The average wages in cell ij for periods s and t thus are defined

$$\bar{w}_{ij}^{s} = \prod_{k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\bar{\omega}_{ijk}^{st}} = \prod_{k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s} + \omega_{ijk}^{t}\right)}$$

and

$$\bar{w}_{ij}^{t} = \prod_{k \in cell_{ij}^{t}} \left(w_{ijk}^{t}\right)^{\bar{\omega}_{ijk}^{st}} = \prod_{k \in cell_{ij}^{t}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s} + \omega_{ijk}^{t}\right)}.$$

We first assume that there are no hires and no separations between periods s and t. Thus the sets of items in periods s and t are the same: $cell_{ij}^{s} = cell_{ij}^{t}$. If we construct the relative of industry × occupation cell ij for an average wage price index as the ratio of average wages,

$$R_{ij}^{AV,st} = \frac{\bar{w}_{ij}^{t}}{\bar{w}_{ij}^{s}} = \frac{\prod_{k \in cell_{ij}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s} + \omega_{ijk}^{t}\right)}}{\prod_{k \in cell_{ij}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s} + \omega_{ijk}^{t}\right)}} = \prod_{k \in cell_{ij}} \left(\frac{w_{ijk}^{t}}{w_{ijk}^{s}}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s} + \omega_{ijk}^{t}\right)} = T^{st};$$

that is, we get the IWPI Törnqvist wage index. Thus, under these conditions, there is no difference between constructing an index as an average of price relatives and as a relative of average prices.[24] If there is a bias in the unadjusted index of employee record wage relatives, there is exactly the same bias in the ratio of (geometric) average employee wages. So averaging

[24] Of course, a difference might be introduced by using arithmetic rather than geometric averaging for the relative of averages method of calculating the cell index, but this is introducing a spurious element into the comparison.

does not eliminate bias due to treating employees as providing the same quality of labor service when they have one additional month of total and employer specific experience, or when there has been a change in their educational attainment, when these materially affect the wage the employee can expect to receive in a given month.

Changes in the assortment of employees (hires and separations) between periods s and t in industry × occupation cell ij complicate this conclusion, because dealing directly with assortment changes introduces the index number problem of new and disappearing items. The standard "matched models" practice is to introduce and link new items into the cell relative after two periods of prices have been recorded. Disappearing items are simply dropped and linked out. Thus, in the matched models index, exactly the same set of employee records is priced in periods s and t. This is the practice followed in the IWPI.

It could be argued that constructing the month to month cell relative as a relative of average wages without matching the employee records between periods s and t somehow addresses the labor quality effect, because the average in the numerator of the relative includes new hires brought in at relatively low wages due to lack of employer specific experience, and separations in the denominator of the relative include experienced, relatively high wage employees exiting the employer's workforce. Thus, the argument is that the ratio of averages relative adjusts price change for employees continuing from period s to t downward to eliminate the slope of the employers' pay scales from wage change. [25] There is heuristic appeal to this argument, because, as we have argued elsewhere, index change resulting from the slope of the pay scale—that is, movement along a given pay scale—is not price change and should be factored out of the index. However, this method is crude. For example, there may be neither hires nor separations between the periods compared, but quality still is changing due to pay scale slope, or there may be hires but no separations, or separations but no hires. Particularly in the first case, the argument that changes in averages are picking up quality creep as employees move up the pay scale loses force. In addition, the direct comparison of the average wages of hires in the current period with the previous period average wages of separations consummated in the current period will include not only quality changes from differences in their positions on the wage scale, which are quality changes, but also price changes from wage scale shift between periods.

The relative of averages between periods when there have been hires and separations, and items comprising the average in the numerator do not match the items comprising the denominator follows the formula:

[25] The case described is a "normal," "good times" case where we imagine separations because of retirements or employees moving on to better opportunities. However, it also is entirely possible that firms having to cut operations separate less experienced staff first, in which case the "assortment" effect works the other way and is *not* related employees' movement along the pay scale. This undermines the mildly favorable argument cited for ratio of averages, in the absence of quality adjusting wages before averaging.

$$R_{ij}^{AV,st} = \frac{\overline{w}_{ij}^{t}}{\overline{w}_{ij}^{s}} = \frac{\prod_{k \in cell_{ij}^{t}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}\right)}}{\prod_{k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}\right)}}$$

where the items in the product in the numerator include hires but exclude separations in period t and the items in the product in the denominator exclude hires but include separations in period t. We therefore can factor this expression as

$$\frac{\prod_{k \in cell_{ij}^{t}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}\right)}}{\prod_{k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}\right)}} = \frac{\prod_{k \in cell_{ij}^{t}\ but\ k \notin cell_{ij}^{s}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}\right)}}{\prod_{k \notin cell_{ij}^{t}\ but\ k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}\right)}} \times \frac{\prod_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}\right)}}{\prod_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}\right)}}.$$

Additionally, we observe that $\omega_{ijk}^{s} = 0$ and $\omega_{ijk}^{t} > 0$ for period t hires and $\omega_{ijk}^{s} > 0$ and $\omega_{ijk}^{t} = 0$ for period t separations, allowing us to simplify the exponential weights in the first factor as

$$\frac{\prod_{k \in cell_{ij}^{t}\ but\ k \notin cell_{ij}^{s}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{t}\right)}}{\prod_{k \notin cell_{ij}^{t}\ but\ k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}\right)}} \times \frac{\prod_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}\right)}}{\prod_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}\right)}}.$$

We now set about identifying a Törnqvist "matched models" cell relative within this expression, with the remaining factor attributable to change in employee assortment between periods s and t. Our first observation is that the exponential weights in the second factor do not add to 1 as required of a matched models Törnqvist index, because the set of items for which $k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}$ is no larger and generally smaller than the set of items for which $k \in cell_{ij}^{t}$ or for which $k \in cell_{ij}^{s}$.

So define the scalars

$$\psi_{ij}^{s} = \left(\frac{\sum_{k \in cell_{ij}^{s}} w_{ijk}^{s} h_{ijk}^{s}}{\sum_{k \in cell_{ij}^{s}\ and\ k \in cell_{ij}^{t}} w_{ijk}^{s} h_{ijk}^{s}}\right) \geq 1$$

and

$$\psi_{ij}^{t} = \left(\frac{\sum_{k \in cell_{ij}^{t}} w_{ijk}^{t} h_{ijk}^{t}}{\sum_{k \in cell_{ij}^{s}\ and\ k \in cell_{ij}^{t}} w_{ijk}^{t} h_{ijk}^{t}}\right) \geq 1,$$

where if there are no hires or separations, $cell_{ij}^{s} = cell_{ij}^{t}$ so that

$$\psi_{ij}^{s} = \psi_{ij}^{t} = 1.$$

We can then rewrite the relative of averages cell index by adding and subtracting $\left(\omega_{ijk}^{*s} + \omega_{ijk}^{*t}\right)$ to and from the exponential weights of its second factor as

$$\frac{\prod_{k \in cell_{ij}^{t}\ but\ k \notin cell_{ij}^{s}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{t}\right)}}{\prod_{k \notin cell_{ij}^{t}\ but\ k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}\right)}} \times \frac{\prod_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{*s}+\omega_{ijk}^{*t}+\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}-\left(\omega_{ijk}^{*s}+\omega_{ijk}^{*t}\right)\right)\right)}}{\prod_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{*s}+\omega_{ijk}^{*t}+\left(\omega_{ijk}^{s}+\omega_{ijk}^{t}-\left(\omega_{ijk}^{*s}+\omega_{ijk}^{*t}\right)\right)\right)}}$$

where

$$\omega_{ijk}^{*s} = \frac{w_{ijk}^{s} h_{ijk}^{s}}{\sum_{k \in cell_{ij}^{s}\ and\ k \in cell_{ij}^{t}} w_{ijk}^{s} h_{ijk}^{s}} = \psi_{ij}^{s} \frac{w_{ijk}^{s} h_{ijk}^{s}}{\sum_{k \in cell_{ij}^{s}} w_{ijk}^{s} h_{ijk}^{s}} = \psi_{ij}^{s} \omega_{ijk}^{s}$$
$$\omega_{ijk}^{*t} = \frac{w_{ijk}^{t} h_{ijk}^{t}}{\sum_{k \in cell_{ij}^{s}\ and\ k \in cell_{ij}^{t}} w_{ijk}^{t} h_{ijk}^{t}} = \psi_{ij}^{t} \frac{w_{ijk}^{t} h_{ijk}^{t}}{\sum_{k \in cell_{ij}^{t}} w_{ijk}^{t} h_{ijk}^{t}} = \psi_{ij}^{t} \omega_{ijk}^{t}.$$

The ratio of weighted geometric averages relative for cell ij becomes

$$\left[\frac{\prod_{k \in cell_{ij}^{t}\ but\ k \notin cell_{ij}^{s}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\omega_{ijk}^{t}\right)}}{\prod_{k \notin cell_{ij}^{t}\ but\ k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\omega_{ijk}^{s}\right)}} \times \frac{\prod_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\left(1-\psi_{ij}^{s}\right)\omega_{ijk}^{s}+\left(1-\psi_{ij}^{t}\right)\omega_{ijk}^{t}\right)}}{\prod_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\left(1-\psi_{ij}^{s}\right)\omega_{ijk}^{s}+\left(1-\psi_{ij}^{t}\right)\omega_{ijk}^{t}\right)}}\right]$$
$$\times \frac{\prod_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \left(w_{ijk}^{t}\right)^{\frac{1}{2}\left(\psi_{ij}^{s}\omega_{ijk}^{s}+\psi_{ij}^{t}\omega_{ijk}^{t}\right)}}{\prod_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \left(w_{ijk}^{s}\right)^{\frac{1}{2}\left(\psi_{ij}^{s}\omega_{ijk}^{s}+\psi_{ij}^{t}\omega_{ijk}^{t}\right)}}.$$

where now

$$\sum_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \psi_{ij}^{s} \omega_{ijk}^{s} = \sum_{k \in cell_{ij}^{t}\ and\ k \in cell_{ij}^{s}} \psi_{ij}^{t} \omega_{ijk}^{t} = 1.$$

The first factor of the ratio of averages relative

$$\left[\frac{\prod_{k\in cell_{ij}^t\ but\ k\notin cell_{ij}^s}\left(w_{ijk}^t\right)^{\frac{1}{2}\left(\omega_{ijk}^t\right)}}{\prod_{k\notin cell_{ij}^t\ but\ k\in cell_{ij}^s}\left(w_{ijk}^s\right)^{\frac{1}{2}\left(\omega_{ijk}^s\right)}}\times\frac{\prod_{k\in cell_{ij}^t\ and\ k\in cell_{ij}^s}\left(w_{ijk}^t\right)^{\frac{1}{2}\left(\left(1-\psi_{ij}^s\right)\omega_{ijk}^s+\left(1-\psi_{ij}^t\right)\omega_{ijk}^t\right)}}{\prod_{k\in cell_{ij}^t\ and\ k\in cell_{ij}^s}\left(w_{ijk}^s\right)^{\frac{1}{2}\left(\left(1-\psi_{ij}^s\right)\omega_{ijk}^s+\left(1-\psi_{ij}^t\right)\omega_{ijk}^t\right)}}\right]$$

is the product of "change in assortment" and a "matched models" scale factor. Notice that, since $\psi_{ij}^s \geq 1$ and $\psi_{ij}^t \geq 1$, the exponents of the scale factor are all negative, so expressing this factor in terms of positive exponents, we can write the scale factor as

$$\frac{\prod_{k\in cell_{ij}^t\ and\ k\in cell_{ij}^s}\left(w_{ijk}^s\right)^{\frac{1}{2}\left(\left(\psi_{ij}^s-1\right)\omega_{ijk}^s+\left(\psi_{ij}^t-1\right)\omega_{ijk}^t\right)}}{\prod_{k\in cell_{ij}^t\ and\ k\in cell_{ij}^s}\left(w_{ijk}^t\right)^{\frac{1}{2}\left(\left(\psi_{ij}^s-1\right)\omega_{ijk}^s+\left(\psi_{ij}^t-1\right)\omega_{ijk}^t\right)}}.$$

Having dealt with the first main factor of the ratio of geometric averages, the second main factor is the Törnqvist "matched models" index of the IWPI

$$\frac{\prod_{k\in cell_{ij}^t\ and\ k\in cell_{ij}^s}\left(w_{ijk}^t\right)^{\frac{1}{2}\left(\psi_{ij}^s\omega_{ijk}^s+\psi_{ij}^t\omega_{ijk}^t\right)}}{\prod_{k\in cell_{ij}^t\ and\ k\in cell_{ij}^s}\left(w_{ijk}^s\right)^{\frac{1}{2}\left(\psi_{ij}^s\omega_{ijk}^s+\psi_{ij}^t\omega_{ijk}^t\right)}}.$$

From this, it is clear that if the average wages of separations in period s are greater than the average wages of hires in period t, the first component of the adjustment factor will be < 1. The second adjustment factor will be < 1 if all wages of continuing employees are increasing. Thus, in this case, which applies to perhaps most industry by occupation cells of the IWPI in Iceland's present economic epoch, the assortment adjustment is less than one, pulling down the price change registered by the "matched models" index when there is a change in assortment. Again, if there are no hires or separations between periods s and t, the adjustment factor is 1 and the ratio of geometric averages relative becomes the Törnqvist "matched models" index of the IWPI.

Assessing the assortment adjustment to the IWPI cell index implied by the ratio of geometric averages index

From this algebraic examination of the ratio of geometric averages cell relative formula, under conditions perhaps prevalent in Iceland's labor market the "matched models" cell relative index will indeed generally show higher rates of price change than the ratio of geometric averages. However, when there is change in assortment from period to period, it is not clear whether the ratio of averages generates the correct assortment adjustment to the matched models index.

First, the economic approach to changes in assortment is to impute a "reservation price" for new and disappearing items—a price at which, for each item contributing to assortment change, the quantity employers demand of that item (the hours worked by an employee if a separation is considered or a potential employee if a hire is considered) is zero. Although index formulae have been worked out for the change in assortment case when the underlying aggregator is either the constant elasticity of substitution (see Feenstra [1994] and Feenstra and Shapiro [2003]) or quadratic aggregator function (Diewert and Feenstra [2018]), this does not seem to have been worked out for the translog aggregator underlying the IWPI Törnqvist cell index in published research. So the assortment factor from the ratio of geometric averages might or might not have the correct functional form.

Second, in view of the incrementing experience effect, even the wages of the employees continuing from month s to t, while controlled for all other price determining characteristics besides education and experience, become noncomparable between periods. So *all* wage relatives are in the "assortment" factor, the scale adjustment and matched models factors disappear, and we are again left with a ratio of geometric averages estimate for wage change.

But we cannot use a ratio of averages of any kind to estimate price change unless the averaging is over items with identical price determining characteristics that do not differ between the periods compared, or the items in the average have different characteristics but are matched on all price determining characteristics between the periods compared.

Quality adjustment standardizes the characteristics of all of the employees in both periods—including their level of experience—and allows us to defend the proposition that all of the employees in periods s and t are alike in every controlled-for respect, including change in education and experience in our employee wage case. In this case, any type of average—the geometric as well as the arithmetic average (unit value)—of quality standardized wages from two periods can be defended as the basis for an index number.

Of course, if standardization has been achieved, then ratio of geometric weighted averages of quality adjusted wages will not materially differ from the Törnqvist matched models index of quality adjusted wages, and the effect of introducing the wages of hires and separations is mainly to reduce variance. Since the percentages of hires and separations within cells for given months is usually small, the quality adjusted ratio of geometric weighted averages, which includes hires and separations, should make very little difference from quality adjusted Törnqvist matched models, which excludes hires and separations.

The key takeaway from this is that it is crucial to quality adjust individual wages within cells when the qualities of item specifications change over time. Adding in hires and separations is of secondary importance, unless this happens to offset the changes in the "average" quality of "matched" items within the cell, leaving the "average" quality of the cell unchanged. But there is an important example where this will not be the case: selective separation of inexperienced employees in an economic downturn.[26]

[26] *Ibid.*

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